Automatic target recognition in SAR images using multilinear analysis

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AUTOMATIC TARGET CLASSIFICATION IN SAR IMAGES USING MPCA

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ABSTRACT

Multilinear analysis provides a powerful mathematical framework for analyzing synthetic aperture radar (SAR) images resulting from the interaction of multiple factors like sky luminosity and viewing angles, while preserving their original shape. In this paper, we propose a multilinear principal component analysis (MPCA) algorithm for target recognition in SAR images. First, we form a high order tensor with the training image set and we apply the higher-order singular value decomposition (HOSVD) to reveal patterns and dependencies between images. The HOSVD of this training image tensor is also used for compressing the data and removing background noise. Then, a multilinear projection algorithm exploiting the calculated HOSVD is used to classify an unknown target in a SAR image. This multilinear projection that leads to a nonlinear optimization problem is carried out in an iterative way by applying the alternate least squares (ALS) algorithm which solves a linear projection subproblem at each iteration. The estimated feature vector associated with the mode-class is then used for recognition. Tests with a true SAR image database illustrate very good classification performance of the proposed MPCA-based method while providing a very high compression rate.

1. INTRODUCTION

Supervised subspace-based classification is a classical approach for pattern recognition. Traditional linear subspace methods like principal component analysis (PCA) or linear discriminant analysis (LDA) require to reshape images into high-dimensional vectors. Then, vectorized images belonging to the same class of objects to classify are ranged into a matrix, and PCA or LDA are applied to determine the basis vectors that span each class. This vectorization breaks the natural structure and correlation in the original image set. Moreover, with linear approaches only single-factor variations are permitted in the image database. During the last decade, several multilinear subspace methods have been proposed for generalizing linear methods, as it is the case of multilinear principal component analysis (MPCA), multilinear independent component analysis (MICA) and multilinear discriminant analysis (MDA) used for face recognition ([1], [2],[3]), handwritten digit classification ([4]), or gait recognition ([5], [6]). These multilinear methods take all the variation factors of the training image database into account by considering it as a high order tensor and determining multiple interrelated subspaces by means of the higher order singular value decomposition (HOSVD) ([7]).

In this paper, we consider the problem of target classification in synthetic aperture radar (SAR) images. Such images are very noisy due to speckle noise. Moreover, unlike optical images, SAR images of a same target observed from different viewing angles are characterized by great variations in appearance, which makes the target classification problem non trivial. Usually, in supervised classification, the test images are expected to belong to predefined training classes. In practical situations, especially for military applications, some test images do not contain objects belonging to learned classes. In this case, two additional classes must be considered : the rejection class and the confusion class. Unlearned targets are labelled into the rejection class while the confusion class is used when the classifier is not able to decide between several classes.

The rest of the paper is organized as follows. In Section II, we recall some definitions relative to tensors. Section III presents the MPCA-based algorithm including training and testing phases of the classification process. A link with two existing MPCA-based classifiers is also established. In Section IV, we show how a SAR image database can be encoded as a fourth-order tensor, and we present some experimental results obtained with true SAR images. In Section V, the data compression ability of our approach is illustrated. Finally, the paper is concluded with some perspective for future work.

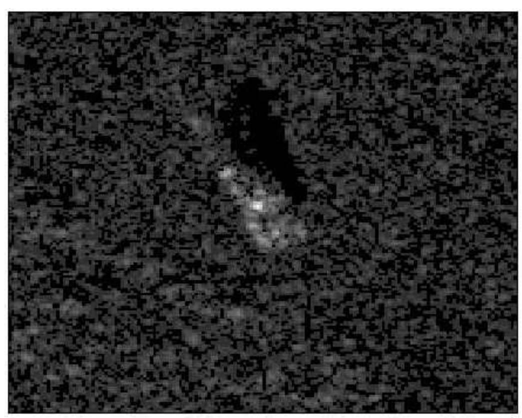

Fig. 1. SAR image of a target in the MSTAR database

Notations : vectors and matrices are represented by boldface lower case ($\mathbf{x}$) and boldface capital ($\mathbf{X}$) letters, respectively, whereas tensors are denoted by blackboard capital letters ($\mathbb{X}$). Rows and columns of matrices are denoted using the matlab notation ; for example, the i-th row of $\mathbf{X}$ is denoted $\mathbf{X}(\mathbf{i}, :)$.

2. TENSOR ANALYSIS

In this section, we recall some definitions relative to tensors and the HOSVD. More details can be found in [8].

2.1. Definitions

A tensor $\mathbb{X}$ of order N can be viewed as an N-way array, of dimensions $I_1 \times I_2 \times \ldots \times I_N$, each entry $x_{i_1 i_2 \ldots i_N}$ being indexed by means of N indices with $1 \leq i_n \leq I_n$. Each index is associated with a way, also called a mode. The mode-n vectors of $\mathbb{X} \in \Re^{I_1 \times I_2 \times \ldots \times I_N}$ are the I_n-dimensional vectors obtained by varying the index i_n while keeping the other indices fixed. The mode-n matricization of $\mathbb{X}$ gives the unfolded matrix $\mathbf{X}_{(n)} \in \Re^{I_n \times I_N \ldots I_{n+1} I_{n-1} \ldots I_1}$ whose columns are the mode-n vectors. The Frobenius norm of a real tensor $\mathbb{X}$ is given by :

$$\|\mathbb{X}\|_F^2 = \sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \ldots \sum_{i_N=1}^{I_N} x_{i_1 \ldots i_N}^2 \quad (1)$$

.

The mode-n product of a tensor $\mathbb{X} \in \Re^{I_1 \times \ldots \times I_n \times \ldots \times I_N}$ with a matrix $\mathbf{A} \in \Re^{J_n \times I_n}$ gives a tensor $\mathbb{Y} \in \Re^{I_1 \times \ldots \times J_n \times \ldots \times I_N}$ whose entries are computed by :

$$y_{i_1 \ldots i_{n-1} j_n i_{n+1} \ldots i_N} = \sum_{i_n=1}^{I_n} x_{i_1 \ldots i_N} a_{j_n i_n} \quad (2)$$

The mode-n product, written in tensor notation as $\mathbb{Y} = \mathbb{X} \times_n \mathbf{A}$, can be expressed in terms of unfolded matrices as $\mathbf{Y}_{(n)} = \mathbf{A}\mathbf{X}_{(n)}$.

2.2. Higher order singular value decomposition (HOSVD)

The HOSVD can be viewed as a high order generalization of the matrix SVD which orthogonalizes the column spaces of the N unfolded matrices $\mathbf{X}_{(n)}, n = 1, \ldots, N$. Such an HOSVD allows to write the tensor $\mathbb{X}$ as ([7]) :

$$\mathbb{X} = \mathbb{S} \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \ldots \times_N \mathbf{U}_N \quad (3)$$

where the matrices $\mathbf{U}_n \in \Re^{I_n \times I_n}, n = 1, \ldots, N$, are orthogonal. The core tensor $\mathbb{S} \in \Re^{I_1 \times \ldots \times I_N}$ defines the interactions between the mode-n matrices $\mathbf{U}_n$ whose columns are orthonormal vectors spanning the column space of $\mathbf{X}_{(n)}$. Frobenius norm obtained in fixing one mode of the core tensor $\mathbb{S}$ are the singular values of the tensor $\mathbb{X} : \sigma_{(i_n,i)} = \|\mathbb{S}_{i_n=i}\|_F$.

3. MPCA-BASED TARGET RECOGNITION

We use MPCA as a supervised algorithm for classification. The training phase consists in forming an $(N+3)^{th}$-order tensor with all images of the database and computing its HOSVD in order to reveal interesting pattern dependencies in the database images. Then, a test image to be classified is projected onto the (N+1) subspaces associated with the first (N+1) modes.

3.1. Training phase

A training set is composed of I_k classes of images characterized by N different variation factors. These factors correspond to the first N modes of the training tensor. Let us consider a database composed of images of size $Y \times X$ belonging to I_k classes characterized by N modes of respective dimensions $I_1, \ldots, I_N$ (we assume that there is the same number of images per class in the training database). Each image of the database is defined by the coordinate set $(i_1, \ldots, i_N, i_k)$ with $i_n = 1, \ldots, I_n, n = 1, \ldots, N$ and $i_k = 1, \ldots, I_k$. The total number of images in the database is given by $(\prod_{n=1}^{N} I_n) I_k$. All the images are ranged into a $(N+3)^{th}$-order tensor $\mathbb{D}$ of dimension $I_1 \ldots \times I_N \times I_k \times X \times Y$ where the last two modes are pixel modes and the $(N+1)^{th}$-mode is the mode-class. HOSVD of this tensor can be written as :

$$\mathbb{D} = \mathbb{S} \times_1 \mathbf{U}_1 \times \ldots \times_N \mathbf{U}_N \times_{N+1} \mathbf{U}_k \times_{N+2} \mathbf{U}_x \times_{N+3} \mathbf{U}_y \quad (4)$$

Each image $\mathbf{D}_{i_1,\ldots,i_n,i_k} \in \Re^{Y \times X}$ belonging to the training database can be written using the HOSVD as follows :

$$\begin{aligned}\mathbf{D}_{i_1,\ldots,i_n,i_k} = \mathbb{S} \times_1 \mathbf{U}_1(i_1,:) \ldots \times_N \mathbf{U}_N(i_N,:) \\ \times_{N+1} \mathbf{U}_k(i_k,:) \times_{N+2} \mathbf{U}_x \times_{N+3} \mathbf{U}_y \quad (5)\end{aligned}$$

Columns of $\mathbf{U}_k$ are singular vectors characterizing energy in the class space. Each row of $\mathbf{U}_k$ is related to one specific class and generates, through the core tensor $\mathbb{S}$, all the images of the i_k^{th} class.

3.2. Testing phase

Let us consider a test image $\mathbf{D}_{test}$. This image can be decomposed using the HOSVD of the training tensor $\mathbb{D}$ as :

$$\mathbf{D}_{test} = \mathbb{S} \times_1 \mathbf{v}_1 \ldots \times_N \mathbf{v}_N \times_{N+1} \mathbf{v}_k \times_{N+2} \mathbf{U}_x \times_{N+3} \mathbf{U}_y \quad (6)$$

where $\mathbf{v}_1, \ldots, \mathbf{v}_N$ and $\mathbf{v}_k$ are unknown row vectors of respective lengths $I_1, \ldots, I_N, I_k$, to be optimized. The optimization problem consists in minimizing the following Frobenius norm under an unit norm constraint on each optimized vector :

$$\begin{aligned}\hat{\mathbf{v}}_1, \ldots, \hat{\mathbf{v}}_N, \hat{\mathbf{v}}_k = \underset{\mathbf{v}_1,\ldots,\mathbf{v}_N,\mathbf{v}_k}{\text{argmin}} ||\mathbf{D}_{test} - \mathbb{S} \times_1 \mathbf{v}_1 \ldots \times_N \mathbf{v}_N \\ \times_{N+1} \mathbf{v}_k \times_{N+2} \mathbf{U}_x \times_{N+3} \mathbf{U}_y||_F^2 \quad (7)\end{aligned}$$

This simultaneous minimization with respect to $\mathbf{v}_1, \ldots, \mathbf{v}_N$ and $\mathbf{v}_k$ leads to a nonlinear optimization problem that can be solved in a suboptimal way by applying the alternate least squares (ALS) algorithm, which amounts to solve a linear projection subproblem at each iteration. Each subproblem is linked to the minimization of a conditional LS cost function associated with a different vectorization of Eq. (6) that can be rewritten in the equivalent element-wise form :

$$\begin{aligned}d_{yx} = \sum_{i_1=1}^{I_1} \ldots \sum_{i_N=1}^{I_N} \sum_{i_k=1}^{I_k} \sum_{i_x=1}^{I_x} \sum_{i_y=1}^{I_y} s_{i_1 \ldots i_N i_k i_x i_y} \\ v_1(i_1) \ldots v_N(i_N) v_k(i_k) u_x(x,i_x) u_y(y,i_y) \quad (8)\end{aligned}$$

Stacking the rows of the image $\mathbf{D}_{test} \in \Re^{Y \times X}$ to form the row vector $\mathbf{d}_{test} \in \Re^{1 \times YX}$, we get the N+1 following vectorized forms of Eq. (6) :

$$\begin{aligned}\mathbf{d}_{test} &= \mathbf{v}_1 \mathbf{S}_{(1)} (\mathbf{U}_y \otimes \mathbf{U}_x \otimes \mathbf{v}_k \otimes \mathbf{v}_N \otimes \ldots \otimes \mathbf{v}_2)^T \\ &= \ldots \\ &= \mathbf{v}_N \mathbf{S}_{(N)} (\mathbf{U}_y \otimes \mathbf{U}_x \otimes \mathbf{v}_k \otimes \mathbf{v}_{N-1} \otimes \ldots \otimes \mathbf{v}_1)^T \\ &= \mathbf{v}_k \mathbf{S}_{(N+1)} (\mathbf{U}_y \otimes \mathbf{U}_x \otimes \mathbf{v}_N \otimes \ldots \otimes \mathbf{v}_1)^T\end{aligned}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{S}_{(n)}$, for $n = 1, \ldots, N+1$, is the mode-n matrix representation of the core tensor $\mathbb{S}$. Each one of these equations can be solved in the LS sense, with respect to one vector factor, conditionally to the knowledge of the N other vector factors, this knowledge being provided first by the initialization and then by estimates obtained at previous iterations. This ALS-based algorithm for computing the multilinear projection is summarized in Table 1 where it and $(.)^{\dagger}$ denote, respectively, the iteration number and the Moore-Penrose matrix pseudo-inverse.

1. Initialization $\hat{\mathbf{v}}_2^{(0)}, \ldots, \hat{\mathbf{v}}_N^{(0)}, \hat{\mathbf{v}}_k^{(0)}, it = 0$
2. Alternate computation
   $\hat{\mathbf{v}}_1^{(it+1)} = \mathbf{d}_{test}(\mathbf{S}_{(1)}(\mathbf{U}_y \otimes \mathbf{U}_x \otimes \hat{\mathbf{v}}_k^{(it)} \otimes \hat{\mathbf{v}}_N^{(it)} \ldots \otimes \hat{\mathbf{v}}_2^{(it)})^T)^{\dagger}$
   $\hat{\mathbf{v}}_1^{(it+1)} \leftarrow \hat{\mathbf{v}}_1^{(it+1)} / \left\| \hat{\mathbf{v}}_1^{(it+1)} \right\|$
   $\ldots$
   $\hat{\mathbf{v}}_N^{(it+1)} = \mathbf{d}_{test}(\mathbf{S}_{(N)}(\mathbf{U}_y \otimes \mathbf{U}_x \otimes \hat{\mathbf{v}}_k^{(it)} \otimes \hat{\mathbf{v}}_{N-1}^{(it+1)} \otimes \ldots \otimes \hat{\mathbf{v}}_1^{(it+1)})^T)^{\dagger}$
   $\hat{\mathbf{v}}_N^{(it+1)} \leftarrow \hat{\mathbf{v}}_N^{(it+1)} / \left\| \hat{\mathbf{v}}_N^{(it+1)} \right\|$
   $\hat{\mathbf{v}}_k^{(it+1)} = \mathbf{d}_{test}(\mathbf{S}_{(N+1)}(\mathbf{U}_y \otimes \mathbf{U}_x \otimes \hat{\mathbf{v}}_N^{(it+1)} \otimes \ldots \otimes \hat{\mathbf{v}}_1^{(it+1)})^T)^{\dagger}$
   $\hat{\mathbf{v}}_k^{(it+1)} \leftarrow \hat{\mathbf{v}}_k^{(it+1)} / \left\| \hat{\mathbf{v}}_k^{(it+1)} \right\|$
3. Return to step 2 until convergence

Table 1. ALS-based multilinear projection

The computation loop is repeated until the difference between $\mathbf{D}_{test}$ and the test image reconstructed using (6) with $(\mathbf{v}_1, \ldots, \mathbf{v}_N, \mathbf{v}_k)$ replaced by their estimated values, becomes smaller than a predefined threshold. The main advantage of this algorithm is its simplicity. However, its convergence is strongly dependent on the initialization. To improve convergence, we propose to use as initial values $\hat{\mathbf{v}}_k^{(0)}$ the row vectors $\mathbf{U}_k(i_k, :)$, with $i_k = 1, \ldots, I_k$, of the mode-N+1 matrix factor of the HOSVD defined in (4). The image $\mathbf{D}_{test}$ is labelled as class i_k^{test} such as :

$$i_k^{test} = \underset{i_k}{\text{argmin}} \left\| \hat{\mathbf{v}}_k - \mathbf{U}_k(i_k, :) \right\|_F^2 \tag{9}$$

Once recognition carried out, all other factors of $\mathbf{D}_{test}$ can be estimated as :

$$i_n^{test} = \underset{i_n}{\text{argmin}} \left\| \hat{\mathbf{v}}_n - \mathbf{U}_n(i_n, :) \right\|_F^2, n = 1, \ldots, N \tag{10}$$

3.3. Comparison with two other MPCA-based recognition methods

Our approach can be viewed as a generalization of two existing MPCA-based recognition methods. In the case of fourth-order tensors ($N = 1$), the I_k LS solutions of (7) obtained for $\mathbf{v}_k = \mathbf{U}_k(i_k, :), i_k = 1, \ldots, I_k$, can be interpreted as the projections of the test image $\mathbf{D}_{test}$ onto I_k sets of basis matrices $\{\mathbf{A}_{i_1}^{(i_k)}, i_1 = 1, \ldots, I_1\}$ characterizing the I_k classes. The classification criterion can then be rewritten as :

$$\begin{aligned} i_k^{test} &= \underset{i_k}{\text{argmin}} \min_{\mathbf{v}_1} \left\| \mathbf{D}_{test} - \mathbb{S} \times_1 \mathbf{v}_1 \times_2 \mathbf{U}_k(i_k, :) \times_3 \mathbf{U}_x \times_4 \mathbf{U}_y \right\|_F^2 \\ &= \underset{i_k}{\text{argmin}} \min_{\mathbf{v}_1} \left\| \mathbf{D}_{test} - \sum_{i_1=1}^{I_1} \mathbf{v}_1(i_1) \mathbf{A}_{i_1}^{(i_k)} \right\|_F^2 \end{aligned}$$

with $\mathbf{A}_{i_1}^{(i_k)} = \mathbb{S}(i_1, :, :, :) \times_2 \mathbf{U}_k(i_k, :) \times_3 \mathbf{U}_x \times_4 \mathbf{U}_y$. The basis matrices $\mathbf{A}_{i_1}^{(i_k)}$ are the same as the ones defined in [4]. Our approach can also be viewed as an unfolded matrix formulation of the algorithm proposed in [1] that consists in solving Eq.(6) as follows :

$$\mathbb{R} = \mathbf{v}_1 \circ \ldots \circ \mathbf{v}_N \circ \mathbf{v}_k = \mathbb{T}^{\dagger} \times \mathbf{D}_{test} \tag{11}$$

with $\mathbb{T} = \mathbb{S} \times_{N+2} \mathbf{U}_x \times_{N+3} \mathbf{U}_y$. Instead of computing the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_N$ and $\mathbf{v}_k$ by means of the ALS algorithm, the multiliner projection problem is solved in computing the best-rank(1,...,1) approximation of the tensor $\mathbb{R}$. However, this approach needs to compute the inverse tensor $\mathbb{T}^{\dagger}$ which is not clearly defined.

3.4. Confusion and rejection classes

When recognition is carried out using (9), we use the distance $R(i_k) = \|\hat{\mathbf{v}}_k - \mathbf{U}_k(i_k, :)\|_F^2$. Confusion and rejection classes can be incorporated by introducing two thresholds : the confusion threshold Δ_c and the rejection threshold S_r. If $R(i_k) - R(i_k^{test}) < \Delta_c$ for all $i_k \in \{1, \ldots, I_k\}$ with $i_k \neq i_k^{test}$, the decision between different classes can be considered as confused. If $R(i_k) > S_r$ for all $i_k \in \{1, \ldots, I_k\}$, it can be concluded that the unknown target does not belong to any class of the training set. Confusion and rejection thresholds are free parameters, chosen by the operator according to the robustness needed for the classification. Increased values of Δ_c and S_r imply a decrease of the classification error rate but also of the recognition rate.

4. APPLICATION TO TARGET RECOGNITION IN SAR IMAGES

4.1. SAR image database tensor

A SAR image database is composed of a set of images of targets with different aspect angles and grazing angles. Generally, images at one grazing angle are chosen for the training set while images at another grazing angle are chosen for the testing set. The aspect angle is considered as a variation factor. In this case, MPCA is applied to a fourth-order tensor characterized by two pixel modes, one aspect angle mode and one target class mode. Fig.2 presents a fourth-order tensor of a SAR database. The HOSVD of this SAR image tensor can be written as :

$$\mathbb{D} = \mathbb{S} \times_1 \mathbf{U}_{\substack{aspect\\angle}} \times_2 \mathbf{U}_{target} \times_3 \mathbf{U}_x \times_4 \mathbf{U}_y \tag{12}$$

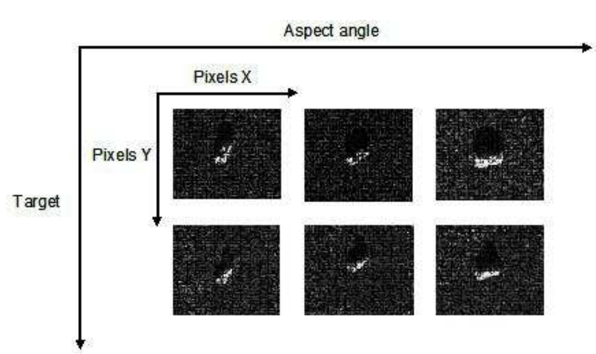

Fig. 2. Fourth-order tensor of MSTAR database images

4.2. Experimental results

Some experiments were carried out using the MSTAR (Moving Stationary Target Acquisition and Recognition) database composed of various images of ten types of targets, with the aspect angle varying between 0˚ and 360˚. Each image corresponds to a target at two different grazing angles : 15˚ and 17˚. In this work, we used a subset composed of five types of targets : BMP2, BRDM2, D7, T62 and T72. Images at 15˚ grazing angle were chosen for the training set while images at 17˚ grazing angle were chosen for the testing set. For both training and testing, we selected a set of 180 images for each target, with an approximative 2˚ aspect angle step. After background masking preprocessing ([9]), images are of dimension 30×50. So MPCA was applied to the fourth-order training tensor of dimension $5 \times 180 \times 30 \times 50$. Recognition (Rc), confusion (Cf) and rejection (Rj) rates obtained after five iterations of the ALS algorithm are given in Table 2. Confusion threshold Δ_c and rejection threshold S_r were chosen empirically to get an apriori fixed recognition rate .

Target	Rc (in %)	Cf (in %)	Rj (in %)
BMP2	83,0	5,4	6,1
BRDM2	89,6	4,3	4,5
D7	95,0	1,6	2,4
T62	81,9	6,2	8,1
T72	81,4	5,6	7,9
Average	86,1	4,6	5,8

Table 2. Recognition, confusion and rejection rates

Rejection performance were evaluated using a target which was not included in the training database. Images of the target 2S1 available in the MSTAR database were considered in the testing phase. With the same thresholds as before, we obtained a rejection rate of 61% for this unlabelled target.

5. DATA COMPRESSION

A tensor-based approach provides a better compression rate than a matrix SVD-based approach. In the matrix case, a low rank approximation is obtained in truncating the SVD. In the same way, a low rank approximation of a tensor can be obtained as a truncated HOSVD, i.e. by discarding the least significant singular values of each unfolded matrix representation of the tensor. Preserving the shape of the data enables a better compression than with a linear approach. For example, after background masking processing, SAR images are sparse because the detection algorithm has removed most of pixels belonging to the clutter. Truncated HOSVD can remove all those null pixels from the data. Fig.3 shows the recognition rate for different compression rates obtained with the MPCA approach and the PCA approach using matrix SVD with vectorisation of database images. In this figure, a 80% compression rate means that we used only 20% of the data volume of the training set to achieve the recognition. As we can see, the MPCA approach allows to improve the recognition rate, comparatively with the PCA approach, from a compression rate of 60%.

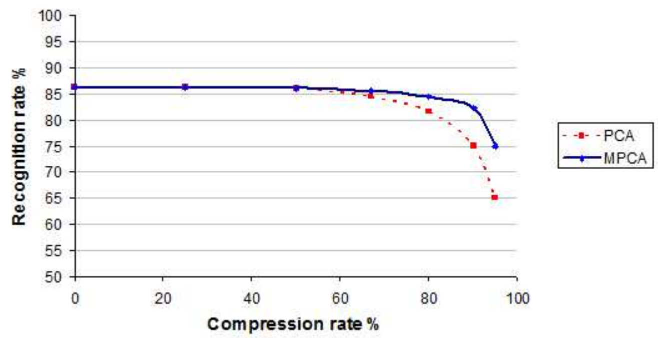

Fig. 3. Recognition rate at different compression rates obtained with MPCA and PCA

6. CONCLUSION

In this paper, a MPCA-based approach has been proposed for automatic target recognition in SAR images. This approach allows to solve a multimodal target recognition problem by considering an image database as a fourth-order tensor. Some simulation results obtained with the MSTAR database illustrate the good performance of the proposed MPCA-based classification method, in terms of both classification and compression rates.

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